Red level

1. **(5 credits)** Mad Hatter wound his two clocks at noon and found that one clock began to lose 2 minutes every hour, and the second one began to gain 1 minute every hour. When Mad Hatter looked at his clocks again, he found that the second clock gained 1 hour in comparison with the first one. At what time did it happen?

Answer: 8 AM
Solution: The difference in clock readings increases by 3 minutes per hour. Therefore, 1 hour difference in clock readings will be accumulated within 20 'real' hours (and if the difference in clock readings is actually not 1 hour, but 13, 25 or even more hours, the 12 hour difference between these values corresponds to another 240 'real' hours, that is exactly 10 days, therefore such aspect does not affect the value in the answer). 20 hours after 12 AM is 8:00 AM the next day.

2. **(7 credits)** Less than 100 passengers were traveling by train in the first coach. There were twice as many standing passengers as sitting ones. 4 percent of passengers got off at the next station. How many passengers remained in the coach?

Answer: 72
Solution: According to the problem statement, the number of passengers in the coach is divisible by 3 (since exactly a third of the passengers was sitting) and by 25 (since 1/25 of all passengers got off at the next station). The only number less than 100 that satisfies both of these conditions is 75. 75 - 3 = 72 passengers remained in the coach after the train left the station.

3. **(8 credits)** Captain Marvel, Superman, Flash and Quicksilver competed in running. When asked who took what place, they gave the following answers:Captain Marvel: 'I was neither the first nor the last one to cross the finish line.' Superman: 'I wasn't the last one.' Flash: 'I was the first one.' Quicksilver: 'I was the last one.' Three of these answers are known to be true and one of them is false. Who was the first to cross the finish line? If Captain Marvel, enter 1, if Superman, enter 2, if Flash, enter 3, if Quicksilver, enter 4.

Answer: Superman (2)
Solution: Three heroes from this list answered that they weren't the last ones. If they all told the truth, then Quicksilver should have the last place, but this means that he also told the truth. So one of the first three heroes lied. This means that Quicksilver told the truth: he was the last, therefore, Superman also told the truth. If Flash told the truth, he was the first, which means that Captain Marvel and Superman also told the truth. Therefore, according to the condition, one boy lied, namely Flash: neither he nor Captain Marvel were the first ones, but Superman was.

4. **(10 credits)** The bisectors of the angles C and D of the trapezoid ABCD split its base AB into three equal parts. Can the diagonal AC be 5 times longer than BD? If yes, enter 1, if no, enter 0.

Answer: No (0)
Solution: It follows from the properties of the angle bisector and the trapezoid that the trapezoid legs are $\frac{2}{3}$ AB or $\frac{1}{3}$ AB. According to the triangle inequality rule, the shorter diagonal is greater than $\frac{1}{3}$ AB, and the longer one is less than 5/3 AB, therefore the ratio of the lengths of the diagonals is less than 5.

5. **(10 credits)** Several friends threw a party at the bar. Every time a woman came in, all the men present drank a glass of absinthe in her honor, and when a man came in, all the women drank a glass of martini when greeting him. What is the minimum number of the participants of the party if the bartender (not a member of this company) served 154 glasses of the drinks to them all?

Answer: 25

Solution: The bartender served only one glass for each pair of friends of different sex (a male friend and a female friend): it was a glass of martini for the female friend, if the male friend came later than her, or a glass of absinthe for the male friend, if he came earlier than her. Therefore 154 is the product of the number of men and the number of women at the party. Since 154 = 2*7*11, he possible products are 1*154, 2*77, 7*22 and 11*14. The last option corresponds to the smallest number of the guests.

6. **(10 credits)** A calculator has two yellow buttons: '+2'and '-2' and two red buttons: 'x3' and '÷3' (the latter only works if a number on the screen is divisible by three.) It is forbidden to press the yellow buttons three times in a row. How many three-digit numbers can be obtained from the input value 2020?

Answer: 450
Solution: Note that all operations with buttons are reversible: if the number X can be obtained from the number Y, then we can obtain Y from X by performing inverse operations. Now we would like to show that we can get the number 2 from any even natural number. At first, we decrease the number by dividing it by 3 (if possible) or by subtracting 2 (or 4), if the number is not divisible by 3. After one or two subtractions, the number becomes a multiple of 3 and can be divided by 3, and therefore it will be possible to get a smaller natural number. It is also clear that all intermediate numbers are even and only the smallest even number can finally be obtained in such a way, that is 2. Since we can get 2 from 2020 and from any even three-digit number, then we can get any even three-digit from 2020. There are 900 three-digit numbers, and exactly half of them are even.

7. **(10 credits)** Several different integers were written in the Professor Moriarty's notebook. Sherlock Holmes found that the product of the two largest of them was 420, and the product of the two smallest of them was half as much as the first one. What is the maximum possible number of such numbers in the notebook?

Answer: 37
Solution: We can get the maximum possible number of such numbers, if all numbers in a row from -15 to +21 were written in the notebook. (The product of the two smallest numbers in such a set is positive and equal to 210.) It is clear that all suitable numbers may be placed in a row between the smallest and largest numbers, so the question resolves itself how large the second largest number can be and how small the second smallest number can be. The second largest number can be estimated as follows: if it is equal to x, then the largest one is greater than x, therefore $x^2 < 420$, and the largest possible integer x = 20. Likewise for the second smallest integer $y^2 < 210$, and the smallest possible integer y = -14.

8. **(12 credits)** How is it possible to place the numbers 2, 3, 4, 7, 8, 9 into the empty boxes to get a correct equality (each digit can be used only once)? Arrange the numbers in the desired order as a single six-digit number in your answer. Specify the largest number of all the possible six-digit numbers.

1/(__ + __) + 5/(__ + __) + 6/(__ + __) = 1

Answer: 428793
Solution: The sum appears to be so small, if fractions with large numerators have large denominators and the denominator of the first fraction is, on the contrary, small. The option with the smallest possible denominator for the first fraction is 1/(2+3), but it is impossible to get the sum of 1 in this case. This is easy to verify by enumerating options where the set 4 + 7 + 8 + 9 is represented as a sum of two summands (11 + 17 or 12+ 16 or 13 + 15: in all these cases, the denominators cannot be canceled and it is impossible to get the sum of 1). Therefore, we have to consider another option, when the first fraction is 1/(2+4). This means that the denominators of the remaining fractions are formed by the components of the set 3 + 7 + 8 + 9, and the only suitable sum is 15 + 12. The first two digits are 4 + 2, the second two digits are 8 + 7, and the third pair of the digits is 9 + 3.

9. (13 credits) The necromancer Vasya placed vertices of the convex hexagon ABCDEF with the sides AB=BC=CD=2 cm and DE=EF=FA=11 cm on a circumference. What is the radius of this circumference?

Answer: 7 cm
Solution: If Vasya rearranges the points on the same circumference in a slightly different order, the 2 and 11 cm sides will alternate. Such hexagon has identical diagonals connecting its next-but-one vertices. This means that all its angles are also identical and equal to 120 degrees. Vasya knows the two sides and the angle between them, therefore he can find the length of this diagonal using the cosine theorem: its square is 2*2+11*11+2*11=147. To find the radius, he can also use the cosine theorem: $R^2 + R^2 + 2R^2 \cos 120 = 147$, from which $R^2 = 49$, R=7.

10. (15 credits) The road between Humpty-City and Dumpty-City goes first along the plain and then along the hillside. The White Knight began his travel from Humpty-City to Dumpty-City, and at the same time the Black Knight began his travel from Dumpty-City in the opposite direction. They met 4.9 km from Humpty-City and then rode on. When they reached their destination cities, they turned and rode back. Their second meeting took place 9.9 km from Humpty-City. Find the distance between the cities, if both knights ride at 15 km/hour along the plain, at 8 km/hour up the hill, and at 24 km/hour down the hill.

Answer: 14 km
Solution: We do not know on which section of the road (the plain or the hillside) the Knights met. Therefore, we have to consider different cases. Let x be the length of the plain section and y be the length of the hillside section.
I. The first meeting took place on the plain section. We make the equation for the time spent on this section from the start to this meeting and get the following: $4.9 / 15 = y / 24 + (x-4.9) / 15$
II. The first meeting took place on the hillside section. Therefore, $x / 15 + (4.9-x) / 8 = (x + y-4.9) / 24$
III. The second meeting took place on the plain section. This means that we can make the equation for the time spent on this section from this meeting to the finish line: $9.9 / 15 = y / 8 + (x-9.9) / 30$.
IV. The second meeting took place on the hillside section. $x / 15 + (9.9-x) / 24 = (x + y-9.9) / 8$
This means that we have to solve an equation system: depending on the case, the system is either I + III, or I + IV, or II + III, or II + IV. We have to check for each option that the result does not contradict the assumptions (in particular, $x < 4.9$ for the equation I, and $x < 9.9$ for equation III). As a result, only the II + IV option is possible, when both meetings took place on the hillside. For this option, x = 4 km and y = 10 km, therefore the distance between the cities is 14 km.