Yellow level

1. **(5 credits)** Pinocchio chose a two-digit number and said the following about it: "If the first digit of my number is even, then the second one is odd." As usual he lied. How many different numbers could he choose in this case?

Answer: 20
Solution: Pinocchio only lies if both digits are even. The mentioned number can be composed by choosing the first digit from 4 options (2, 4, 6, 8), and the second digit from 5 options (0, 2, 4, 6, 8). Total number of options is 4*5=20.

2. **(5 credits)** The positive numbers a, b and c are such that $a2 + 2bc = b2 + c2$ and their sum is 2020. What is the largest of the numbers a, b and c?

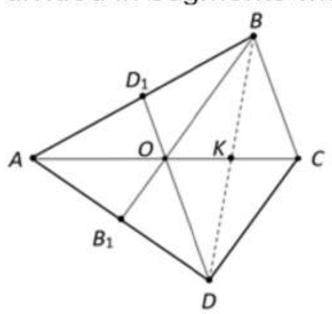
Answer: 1010
Solution: This equation is equivalent to the equation $a2 = (b – c)2$ or $(a + b – c) (a – b + c) = 0$, from which $c = a + b$ or $b = a + c$. This means that the largest number is half the sum of all three numbers, therefore it is equal to 2020:2=1010.
Note: If only b or only c is equal to the sum of other two numbers, this means that the proof is incorrect.

3. **(7 credits)** In how many ways can asterisks in the record 2 *** 2 be arranged so that the resulting number is a multiple of 24?

Answer: 8
Solution1: *(without the divisibility test).* The difference between such adjacent numbers must be divisible by 24 and by 10 (since the last digit is the same); this means that it is equal to LCM (24, 10) = 120. The minimum suitable number is 2112, therefore, other solutions are 2232, 2352, 2472, 2592, 2712, 2832, 2952 (8 numbers in total).

4. **(7 credits)** A ray from the vertex B of the quadrangle ABCD divides the segments AC and AD in half. A ray from the vertex D divides the segments AC and AB in half. In which ratio do the diagonals of the quadrangle divide each other?

Answer: BD is divided in half, AC is divided in segments with lengths in the ratio 3 : 1.



Solution: At first we designate the points as shown in the figure. The line BB1 is parallel to the line CD (according to the problem statement, OB1 is the line joining midpoints of two sides of the triangle ACD). Similarly, we see from the triangle ABC that DD1 and BC are parallel. Therefore, BCDO is a parallelogram. This means that the point K is the midpoint of OC and that it is the answer to the problem.

5. **(7 credits)** The sum of some natural number A and the sum of its digits is equal to B. The sum of B and the sum of its digits is equal to C. Then we got the initial number A after subtracting the sum of digits of the number C from the number C. What is the minimum value of A matching the problem statement?

Answer: 81
Solution: If the sum of digits of a number is subtracted from this number, then the result will be a multiple of 9. Therefore, the number A must be divisible by 9. The sums of digits of the first ten such numbers are equal to 9; 99 is the only number whose sum of digits is greater than 9. Therefore, the smallest operand which is at the same time the result of the three mentioned operations is 81.

6. (10 credits) Is it possible to place the numbers 1, 2,…, 16 into cells of the 4 x 4 table so that any two numbers in its horizontally or vertically adjacent cells differ by 2 or 3? If yes, enter 1, if no, enter 0.

Answer: No (0)
Solution: The number 1 is in the corner cell, since only 3 and 4 can be placed in adjacent cells. The number 2 is also in the corner cell, since only 4 and 5 can be placed in adjacent cells. These corner cells must have a common adjacent cell with the number 4, which is impossible.

7. (12 credits) How many ways can the word CANADA be 'read', if we start from a certain cell and move horizontally or vertically from the letter in this cell to the letter in any adjacent cell? Any letter may be read several times.

				C
			C	A
		C	A	N
	C	A	N	A
C	A	N	A	D

Answer: 32
Solution: The word CANAD can be read from D to C in 2*2*2*2=16 ways, and we can move from D to A in 2 ways. 16*2=32 ways in total.

8. (12 credits) Find the common fraction with the denominator less than 17, which is located on the coordinate line between 15/17 and 17/19.

Answer: 8/9
Solution: It can be shown that this fraction is unique.

9. (15 credits) In the triangle ABC, the angle A is 46°, and the angle C is 78°. O is a point on the bisector of the angle B inside the triangle; this point is placed so that the angle AOC is 118°. Find all angles in the triangle AOC.

Answer: A=23, C=39, O=118
Solution: If O is the center of the inscribed circle, then the situation satisfies the problem statement. If point O is closer to the vertex B than the center of the inscribed circle, then the angle AOC is smaller than 118°, if it is farther from this vertex then the mentioned angle is greater than 118°. If O is the center of the inscribed circle, then AO and CO are bisectors of the angles of the triangle. This means that the required angles are 46:2 = 23 degrees and 78:2 = 39 degrees.

10. (20 credits) A rectangle n × m is cut into three-cell corners. The total length of the cuts is 2011. Find the lengths of the sides of the rectangle.

Answer: 4 и 465
Solution: The length of the cuts is 2mn-m-n-2mn/3=2011, therefore after the transformation 4mn-3m-3n=6033 we get the equation (4n-3)(4m-3)=24141.=13 * 1857. 4n-3=13, 4m-3=1857, and this gives us the answer.