Green Level

1. (**5 credits**) There were several raspberries on the plate. Paddington Bear ate half of all raspberries and another 17 raspberries and threw one remaining spoiled raspberry into the trash can. How many raspberries did Paddington eat?

Answer. 35 raspberries

Solution. After Paddington ate half of all raspberries, exactly half of all cherries remained on the plate. Paddington ate 17 raspberries and threw one raspberry into the trash can from that remainder, i.e. there were 18 raspberries in the plate at this stage, therefore 18 is half of the total amount. This means that Paddington ate 18+17=35 raspberries.

2. (**5 credits**) Mary Poppins has 5 unused vacation days. Weekends and holidays are marked red (in the frame) on Fig. 1. What is the maximum number of subsequent calendar days of rest she may have?

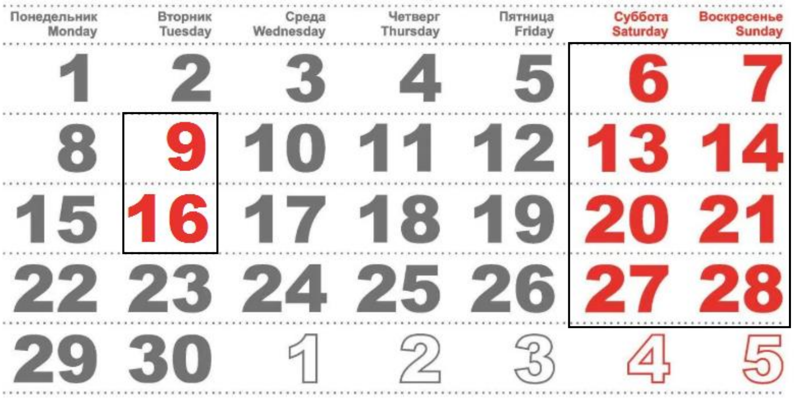


Fig. 1

Answer. 11 days

Solution. If Mary takes a vacation comprising the 8th, 10th, 11th, 12th and 15th days of the month, her vacation lasts from the 6th day to the 16th day, i.e. 11 days. This interval includes 4 weekend days and 2 holidays. The maximum number of weekend days which may be included in the vacation is 4, because there are five days between weekends. Accordingly, with any other combination of days one holiday (or more) is not included and the vacation period reduces.

3. (**7 credits**) Each Martian has five hands. A kindergarten group of Martian children lines up in pairs for a walk (see Fig. 2). According to the rules of the kindergarten, each child has to take the hand of each his neighbour on the left, on the right, behind or in front of him or her. How many hands of children of that group remain free?

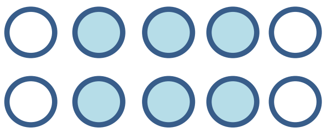


Fig. 2

Answer. 24 hands remain free.

Solution. Please note that children standing inside the columns (they are marked grey in Fig. 2) have two free hands each, because each of them has three neighbours. Other children have two neighbours, and this means that they have three free hands each. Therefore, the total number of the free hands is $6 \cdot 2 + 4 \cdot 3 = 24$.

4. (**7 credits**) Artemis Fowl wrote all consecutive numbers from 1 to 50 in a row without intervals. Then he found five most frequent digits in this sequence and crossed out all such digits. What digit stands in the new sequence in the first place?

   Answer. 6.

   Solution. Each digit of ones occurs only once in each ten. Therefore, the difference in the number of digits used can only be due to digits of tens, i.e. 1, 2, 3 and 4, because they are most frequent. Besides of that, the last ten contains 5, which is the fifth most frequent digit. This means that only digits 6, 7, 8, 9, 0 remain after striking out.

5. (**10 credits**) Watson told Holmes the double-digit number of his apartment. Holmes discovered that this number multiplied by 3 gives a two-digit number. Then Holmes subtracted 3 from the original number,

divided the difference by 3 and finally got a two-digit number as well. What number did Watson tell Holmes?

Answer. 33.

Solution. If a two-digit number still consists of two digits after multiplying by 3, then it is equal to or less than 33. If a two-digit number still consists of two digits after dividing by 3, then the result of subtracting 3 from the original number is equal to or more than 30, i.e. the original number is equal to or more than 33.

6. (**10 credits**) Each of little students of the school on the small island of Komodo got 4 cards containing syllables KO, MO or DO. The cards were distributed so that 13 students can make up the word DODO, 15 students can make up the word KOKO, and 17 students can make up the word MOMO. Moreover, 45 students can make up the word KOMODO using their cards. What is the total number of the students?

Answer: 45 students.

Solution. Children who can make up the word COMODO use 3 cards for that purpose, the remaining 4th card is the same as one of three cards used to make up the word COMODO. Therefore, any student able to make up the word COMODO is also able to make up the word DODO or COCO or MOMO. But 45 = 13 + 15 + 17. Each child has at least 2 identical cards, i.e. each student is able to make up the word DODO or COCO or MOMO; therefore, all students have three different cards and all of them can make up the word COMODO, i.e. the number of the students is 45.

7. (**13 credits**) Sons of Baron Munchausen always lie, sons of Baron Porthos always tell the truth. Sons of both Barons live in the same tavern. Three of them met in the hall.

The first one said, 'I am the only such Baron here.'
The second one said, 'Yes, he is the only such Baron here.'
The third one said, 'Yes, there is only one Munchausen in this hall.'

Whose sons are they?

Answer: All of them are sons of Baron Munchausen.

Solution. The first two men say the same thing. Therefore, they both lie or both tell the truth. If the first one tells the truth, then the second one tells the truth as well. This means that they both are sons of Baron Porthos, which leads to a contradiction. Therefore, they both lie, i.e. they both are sons of Baron Munchausen. This means that the third one also lies and that all of them are sons of Baron Munchausen.

8. (**13 credits**) A line drawn across a paper triangle divided its area in half. Then the triangle was folded along this line so that the area of the "double-layer part" (marked gray in Fig. 3) was equal to the area of the "single-layer part" and was 12 cm^2 smaller than the area of the original triangle. Find the area of the little triangle below the dashed line.

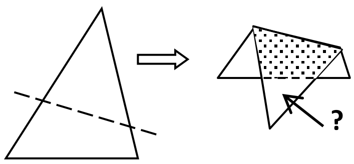
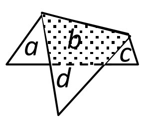


Fig. 3

Answer. 3 cm^2

Solution 1. Let us denote parts of the triangle *a, b, c* and *d*. It follows from the first condition that $a + b + c = b + d$, or $a + c = d$. It follows from the second condition that $b = a + c + d$, i.e. $b = 2d$. It also follows from the second condition that $a + c + d = a + 2b + c + d - 12$ (both parts are equal to b), from which $b = 6$. Therefore, $d = 3$.



Solution 2. When the triangle was folded, its area was reduced by the area of the "double-layer part". Therefore, the area of the grey part is equal to 12. Since the triangle was split into two parts having equal areas, the area of each part is equal to the area of the same grey figure + one or two triangles. I.e. the

area of the triangle below the dashed line is equal to the area of two other triangles, and the sum of their areas is equal to the area of the grey figure.

9. (**15 credits**) The game "Liar's Draughts" has the following rules. Players put dark or light pieces on the board in turns. If a player puts a light piece, he has to tell the truth, and if he puts a dark piece, he has to lie. There is 1 piece on the board. Pinocchio put one more piece and said, 'Now the number of dark pieces on the board exceeds the number of light ones.' What is the colour of the first piece?

***Answer*.** Light.

Solution. Please note that Pinocchio could not put a light piece in any case, because the number of dark pieces could not be increased by putting a light piece regardless of colour of the first piece. This means that Pinocchio put a dark piece and had to lie. Therefore, a dark piece could not be put on the board, because in this case Pinocchio told the truth.

10. (**15 credits**) Kai divided a large rectangle ice plate into 9 different small rectangles with two segments parallel to one side of the plate and two segments parallel to the other side (see Fig. 4). Gerda wants to know the sum of the perimeters of all 9 small rectangles and may ask Kai about the perimeter of any small rectangle in a single question. What is the minimum number of questions giving her the possibility to know the sum?

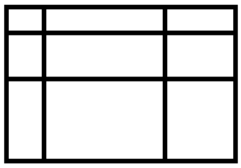


Fig. 4

Answer: 3 questions.

Solution. Please note that the sum of perimeters of all 9 rectangles is equal to the perimeter of the original rectangle multiplied by 3. Therefore, we have to find the perimeter of the whole rectangle. Please also note that the task can be solved by finding perimeters of three rectangles. See Fig. 5 showing the solution.

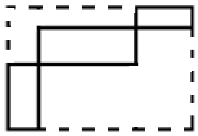


Fig. 5

Let us prove that it is not enough to know perimeters of any two rectangles. Indeed, if any two rectangles are selected, this means that any horizontal segment of three segments is not taken into consideration. For example, it is the middle segment in Fig.6. But this means that having the two selected rectangles unchanged we can get different perimeters of the original rectangle by changing the middle segment.

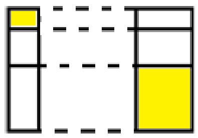


Fig. 6